

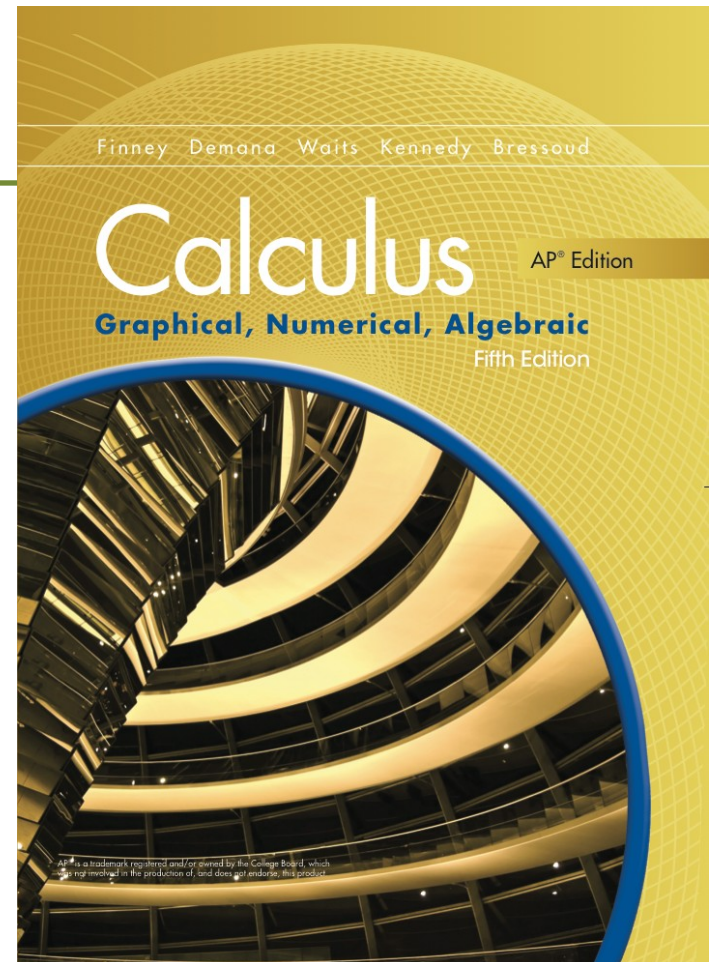
Chapter 5

Applications of Derivatives



Section 5.4

Modeling and Optimization



Quick Review

1. Use the first derivative test to identify the local extrema of

$$y = x^3 - 6x^2 + 12x - 9.$$

2. Use the second derivative test to identify the local extrema of

$$y = 2x^3 + 3x^2 - 12x - 1$$

3. Find the volume of a cone with radius 4 cm and height 7 cm.

Rewrite the expression as a trigonometric function of the angle θ .

4. $\sin(-\theta)$

5. $\cos(-\theta)$

6. Use substitution to find the exact solution of the following system of equations.

$$\begin{cases} x^2 + y^2 = 4 \\ y = \sqrt{3}x \end{cases}$$

Quick Review Solutions

1. Use the first derivative test to identify the local extrema of

$$y = x^3 - 6x^2 + 12x - 9. \quad \text{none}$$

2. Use the second derivative test to identify the local extrema of

$$y = 2x^3 + 3x^2 - 12x - 1 \quad \text{Local maximum: } (-2, 19); \text{ Local minimum } (1, -8)$$

3. Find the volume of a cone with radius 4 cm and height 7 cm. $\frac{112\pi}{3} \text{ cm}^3$

Rewrite the expression as a trigonometric function of the angle θ .

4. $\sin(-\theta) = \sin\theta$

5. $\cos(-\theta) = \cos\theta$

6. Use substitution to find the exact solution of the following system

of equations. $(1, \sqrt{3}); (-1, -\sqrt{3})$

$$\begin{cases} x^2 + y^2 = 4 \\ y = \sqrt{3}x \end{cases}$$

What you'll learn about

- Development of a mathematical model
- Analysis of the function used to model the situation
- Identification of critical points and endpoints
- Identification of solution
- Interpretation of solution

...and why

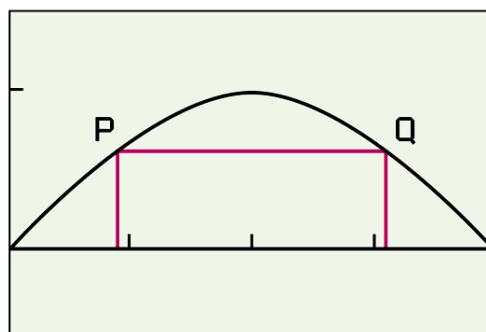
Historically, optimization problems were among the earliest applications of what we now call differential calculus.

Strategy for Solving Max-Min Problems

- 1. Understand the Problem** Read the problem carefully. Identify the information you need to solve the problem.
- 2. Develop a Mathematical Model of the Problem** Draw pictures and label the parts that are important to the problem. Introduce a variable to represent the quantity to be maximized or minimized. Using that variable, write a function whose extreme value gives the information sought.
- 3. Graph the function** Find the domain of the function. Determine what values of the variable make sense in the problem.
- 4. Identify the Critical Points and Endpoints** Find where the derivative is zero or fails to exist.
- 5. Solve the Mathematical Model** If unsure of the result, support or confirm your solution with another method.
- 6. Interpret the Solution** Translate your mathematical result into the problem setting and decide whether the result makes sense.

Example Inscribing Rectangles

A rectangle is to be inscribed under one arch of the sine curve. What is the largest area the rectangle can have, and what dimensions give that area?



$[0, \pi]$ by $[-0.5, 1.5]$

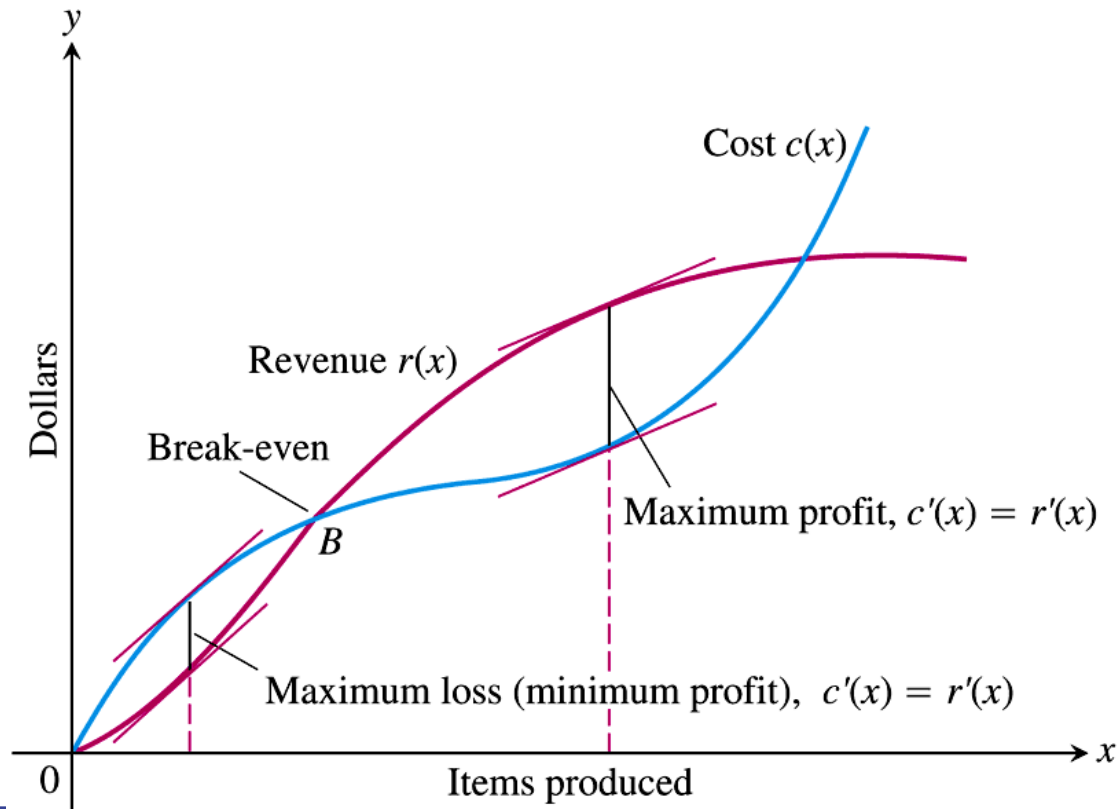
Let $(x, \sin x)$ be the coordinates of point P and the x -coordinate of Q is $(\pi - x)$.

Thus $\pi - 2x = \text{length of rectangle}$ and $\sin x = \text{height of rectangle}$.

$A(x) = (\pi - 2x) \sin x$ where $0 \leq x \leq \pi / 2$. Notice that $A(0) = A(\pi / 2) = 0$. Find the critical values: $A'(x) = -2\sin x + (\pi - 2x) \cos x$. Let $A'(x) = 0$ and use a graphing calculator to find the solution(s). $A'(x) = 0$ at $x \approx 0.71$. The area of the rectangle is $A(0.71) = 1.12$, where the length is 1.72 and its height is 0.65.

Maximum Profit

Maximum profit (if any) occurs at a production level at which marginal revenue equals marginal cost.



Example Maximizing Profit

Suppose that $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of units. Is there a production level that maximizes profit? If so, what is it?

Set $r'(x) = 9$ equal to $c'(x) = 3x^2 - 12x + 15$.

$$3x^2 - 12x + 15 = 9$$

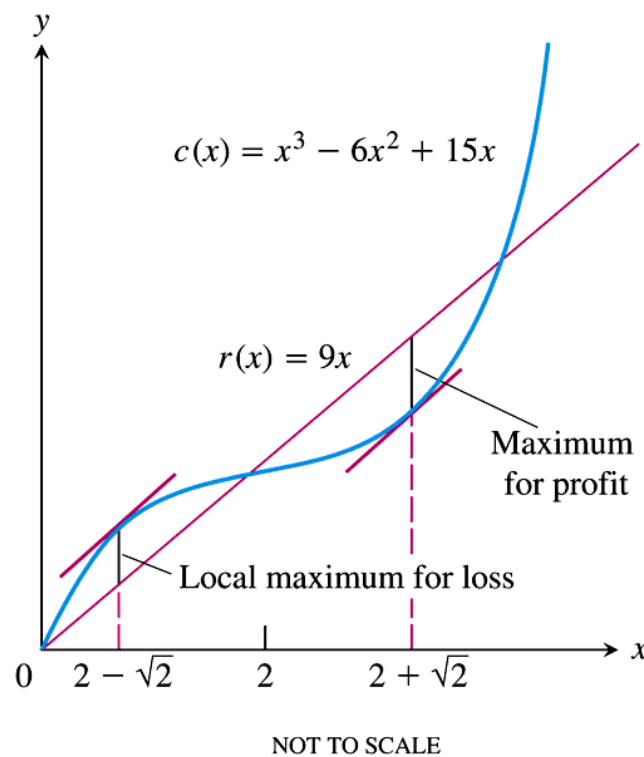
$$3x^2 - 12x + 6 = 0$$

Use the quadratic equation to find

$$x_1 = 2 - \sqrt{2} \approx 0.586$$

$$x_2 = 2 + \sqrt{2} \approx 3.414$$

Use a graph to determine that the maximum profit occurs at $x \approx 3.414$.



Minimizing Average Cost

The production level (if any) at which average cost is smallest is a level at which the average cost equals the marginal cost.